

05 Math I:

- 1) let x be number of 2 pence coins
 let y be number of 5 pence coins

$$\begin{aligned} x + y &= 36 & \Rightarrow 2x + 2y &= 72 \\ 2x + 5y &= 129 & 2x + 5y &= 129 \\ & & -3y &= -57 \\ & & y &= 19 \\ & \therefore x &= 36 - 19 = 17 \end{aligned}$$

17 5 pence coins

- 2) $\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \dots$

a) Add numerator & denominator in previous to get new denominator

• Add all numbers of 2 previous fractions to get numerator

$$\begin{aligned} b) 7 + 17 + 5 + 12 &= 41 \\ 17 + 12 &= 29 \\ 41 + 29 + 17 + 12 &= 99 \\ 41 + 29 &= 70 \end{aligned}$$

$$\frac{41}{29}, \frac{99}{70} \quad \text{Next} = \frac{239}{169}$$

- c) $1, 1.5, 1.4, 1.417, 1.414, 1.414$

converges to 1.414

$$3) P = c\pi^3$$

H

$$a) P = \frac{12.7(\pi)^3}{5.2}$$

$$= 75.727$$

$$b) 9.3 = c\pi^3$$

$$17.3$$

$$160.89 = c\pi^3$$

$$c = 5.189$$

$$c) 62 = \frac{30\pi^3}{15}$$

$$930 = 30\pi^3$$

$$31 = \pi^3$$

$$\pi = 3.141380652$$

4)

a) $1 = \frac{1}{2} + \frac{1}{3}$ • largest unit fraction less than 1 is $\frac{1}{2}$
 • next largest is $\frac{1}{3}$

$$1 = \frac{1}{2} + \frac{1}{3} + \dots = \frac{5}{6} + \dots$$

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

b) Multiple 1 by $\frac{1}{6}$

$$\Rightarrow \frac{1}{6} \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \right) = \frac{1}{12} + \frac{1}{18} + \frac{1}{36}$$

$$c) 1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{12} + \frac{1}{18} + \frac{1}{36}$$

$$\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}$$

$$d) 1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{12} + \frac{1}{19} + \frac{1}{342} + \frac{1}{37} + \frac{1}{1332} \quad \frac{1}{3} = \frac{1}{4} + \frac{1}{3(4)}$$

$$\frac{1}{2} + \frac{1}{2}$$

$$= \frac{1}{2} + \left(\frac{1}{2} - \frac{1}{3} \right) + \frac{1}{3}$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \left(\frac{1}{6} - \frac{1}{7} \right)$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{42}$$

$$= \frac{1}{2} + \frac{1}{4} + \left(\frac{1}{3} - \frac{1}{4} \right) + \frac{1}{7} + \frac{1}{42}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{12} + \frac{1}{42}$$

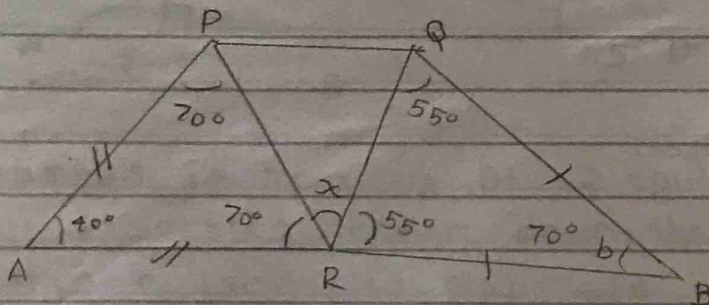
$$= \frac{1}{2} + \left(\frac{1}{4} - \frac{1}{5} \right) + \frac{1}{5} + \frac{1}{7} + \frac{1}{12} + \frac{1}{42}$$

$$= \frac{1}{2} + \frac{1}{5} + \frac{1}{7} + \frac{1}{12} + \frac{1}{20} + \frac{1}{42}$$

$$= \frac{1}{2} + \left(\frac{1}{5} - \frac{1}{6} \right) + \frac{1}{6} + \frac{1}{7} + \frac{1}{12} + \frac{1}{20} + \frac{1}{42}$$

$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{7} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42}$$

5)



$$a) a = 40^\circ, b = 70^\circ$$

$$\angle APR = 70^\circ, \angle ARP = 70^\circ$$

$$\angle BRQ = 55^\circ, \angle BQR = 55^\circ$$

$$x = 180 - 70 - 55 = 55^\circ$$

$$7) y = \frac{3}{\sqrt{1-x^2}}$$

$$a) y = \frac{3}{\sqrt{1-0.2^2}} = 3.061 = 3.06$$

$$b) x=0: y = \frac{3}{\sqrt{1}} = 3$$

$$x=0.2: \frac{3}{\sqrt{1-0.2^2}} = 3.06$$

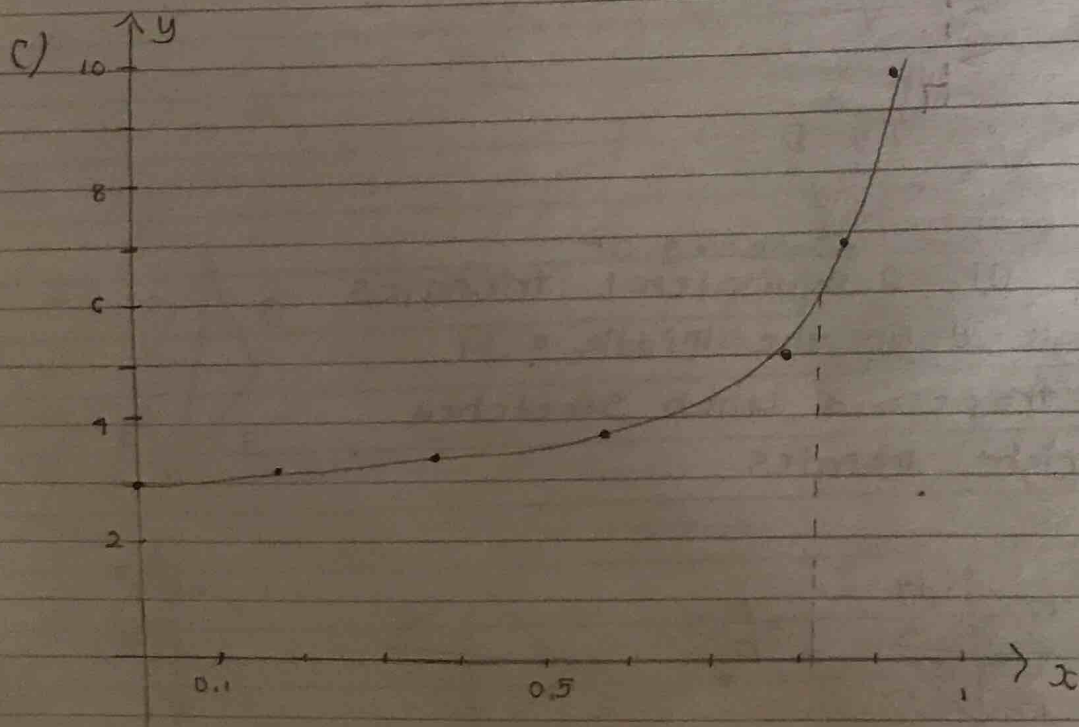
$$x=0.4: \frac{3}{\sqrt{1-0.4^2}} = 3.27$$

$$x=0.6: \frac{3}{\sqrt{1-0.6^2}} = 3.75$$

$$x=0.8: \frac{3}{\sqrt{1-0.8^2}} = 5$$

$$x=0.9: \frac{3}{\sqrt{1-0.9^2}} = 6.88$$

$$x=0.95: \frac{3}{\sqrt{1-0.95^2}} = 9.61$$



$$d) x = 0.866$$

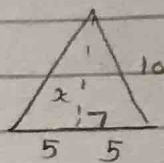
e) Not exponentially shaped (curve).

8)

$$a) A: \text{Area} = \frac{60 \times \pi r^2}{360} = \frac{\pi r^2}{6}$$

$$B: \text{Area} = \frac{1}{2} (10)(\sqrt{75})$$

$$= 5\sqrt{75}$$



$$5^2 + x^2 = 10^2$$

$$x^2 = 100 - 25$$

$$x = \sqrt{75}$$

$$\frac{\pi r^2}{6} = 5\sqrt{75}$$

$$\pi r^2 = 30\sqrt{75}$$

$$r = \frac{\sqrt{30\sqrt{75}}}{\sqrt{\pi}}$$

$$r = 9.09$$

$$b) A: \text{Perimeter} = r + r + \left(\frac{60 \times 2\pi r}{360} \right)$$

$$= r + r + \frac{1 \times 2\pi r}{6}$$

$$= r + r + \frac{\pi r}{3}$$

$$= 2r + \frac{\pi r}{3}$$

$$B: \text{Perimeter} = 30$$

$$\frac{2r + \pi r}{3} = 30 \Rightarrow r \left(\frac{2 + \pi}{3} \right) = 30 \Rightarrow r = \frac{30}{\frac{2 + \pi}{3}} = 9.85$$

$$9) F_1 = 1, F_2 = 1, F_3 = 2$$

$$a) F_4 = 2 + 1 = 3$$

$$F_5 = 3 + 2 = 5$$

$$F_6 = 5 + 3 = 8$$

$$F_7 = 8 + 5 = 13$$

$$F_8 = 13 + 8 = 21$$

$$F_9 = 21 + 13 = 34$$

$$F_{10} = 34 + 21 = 55$$

$$b) F_1^2 + F_2^2 = 2 = F_3$$

$$F_2^2 + F_3^2 = 1 + 4 = 5 = F_5$$

$$F_3^2 + F_4^2 = 4 + 9 = 13 = F_7$$

$$F_4^2 + F_5^2 = 9 + 25 = 34 = F_9$$

$$F_5^2 + F_6^2 = 25 + 64 = 89 = F_{11}$$

If you add the subscripts of F together you get the term. (ie $F_1^2 + F_2^2 = F_3 \Rightarrow 1+2=3$)

$$i) c) F_{50}^2 + F_{51}^2 = F_{51+50} = F_{101}$$

$$ii) F_4^2 + F_{4+1}^2 = F_{137}$$

$$4 + 4 + 1 = 137$$

$$24 = 136$$

$$4 = 68$$